Comments on the internal motion of massive charm quark in the process of double charmonium production in e^+e^- annihilations

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The cross section value of doubly charmonium production in e^+e^- annihilations have been estimated at interaction energy $\sqrt{s} = 10.6$ GeV. The quarkonium wave function shape, as well as the nonzero value of charm quark have been taken into account.

The recent experimental results of the BELLE and BABAR Collaborations on doubly charmonium in e^+e^- annihilations demand an essential revision of the calculation techniques based on QCD factorization theorem. Indeed, the cross section value of charmonium pair production in e^+e^- annihilations estimated within standard calculation technique [1] underestimates the experimental data by an order of magnitude [2, 3]. The detailed analysis has shown that one of the reasons of such underestimation are large fixed virtualities of the intermediate quark and gluon $q^2 \sim \frac{Q^2}{4}$ (Q^2 is a virtuality of the initial photon, see fig. 1), which occur in the standard calculations, where relative momenta of valence quarks of charmonium are not taken into account in the hard part of amplitude (the so-called δ-approximation).

Taking into account the relative motion of valence quarks allows to decrease contradictions between theory and experiment. Calculations of the pair quarkonium production in e^+e^- annihilations, obtained within a light cone formalism [4, 5, 6] are in a qualitative agreement with the experimental data of the BELLE and BABAR Collaborations. In these works, the δ-shaped wave function of quarkonium was replaced by a function $\phi(x)$, which is "spread" on x, where x is a momentum fraction of the quarkonium carried by a valence quark in the infinite momentum frame. Thus, the longitudinal "internal motion" of the valence quarks is taken into account (see also [7]). This spreading decreases the effective virtuality of the intermediate quark and gluon, and, therefore, increases the predicted cross section value.

An analysis of doubly charmonium production within quark-hadron duality also lead to the cross section value increase [8], and allows to describe the data.

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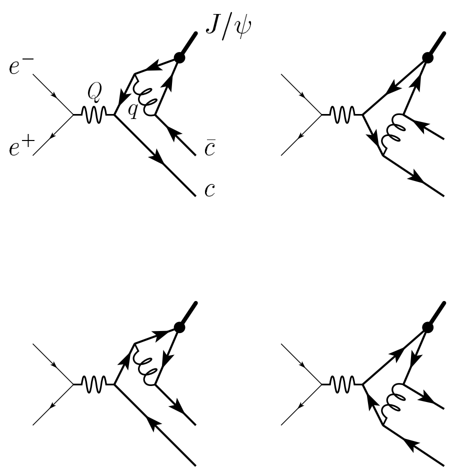

Figure 1: The leading order diagrams for the process $e^+e^- \rightarrow J/\psi + c\bar{c}$. Q is momentum of the photon and q is momentum of the intermediate quark or gluon. The average of q^2 in the total phase space is about several m_c^2. If both $c\bar{c}$-pairs are fused into charmonia ($e^-e^+ \rightarrow J/\psi +$ charmonium), then $q^2 \sim Q^2/4$ within the δ-approximation.

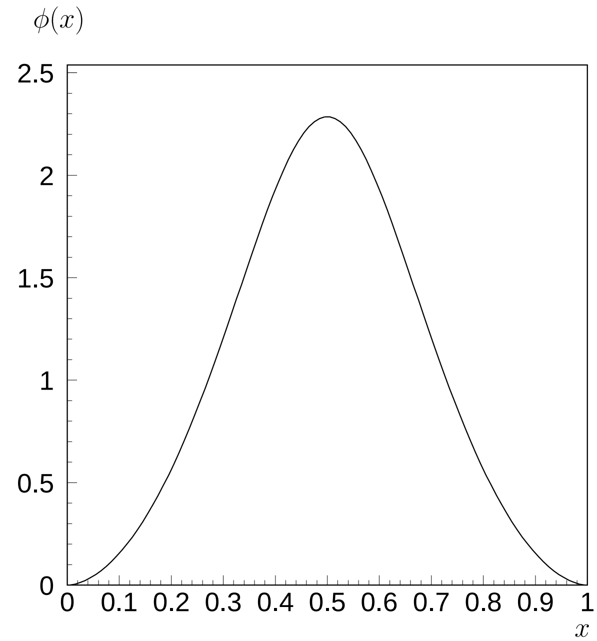

Figure 2: The shape of charmonium wave function used in [4] within the lightcone formalism (leading twist).

Recently several works appear, where the internal motion effect has been estimated in the framework of NRQCD [9, 10]. The received v^2 corrections increase the cross section by factor about $1.5 - 2.5$. This value is less than value ~ 10 obtained within lightcone approximation [4, 5, 6]. Also there is a study [11], where the critical analysis of lightcone formalism has been done for the double charmonium production in e^+e^- annihilations. The predictions obtained in this work are close to the NRQCD estimations [9, 10].

In this work we study the dependence of internal motion effect on the wave function shape, as well as on the c quark mass value. For this purpose we use a simple model described in [12] instead of rigorous lightcone formalism or NRQCD. Within the model under consideration an amplitude of $J/\psi\eta_c$ production in e^+e^- annihilations is performed as follows:

$$A_{J/\psi\eta_c} = \iint dx\, dy\, \phi_{J/\psi}(x)\phi_{\eta_c}(y)T_{c\bar{c}c\bar{c}}, \tag{1}$$

where $\phi_{J/\psi}(x)$ is a wave function of J/ψ, x is p_+ fraction of c quark inside J/ψ, $\phi_{\eta_c}(y)$ is a wave function of η, y is p_+ fraction of c quark inside η_c, and $T_{c\bar{c}c\bar{c}}$ is a hard part of amplitude, which describes the process of four charm quark production. $c\bar{c}$-pairs in $T_{c\bar{c}c\bar{c}}$ have quantum numbers corresponded to J/ψ and η_c states. The transverse motion of quarks inside charmonium is neglected.

For $\phi_{J/\psi}$ and ϕ_{η_c} functions one choose the parametrization used in work [4] within the lightcone formalism:

$$\phi_{lightcone}(x, v^2) = c(v^2)\phi^a(x) \left[\frac{x(1-x)}{1-4x(1-x)(1-v^2)}\right]^{1-v^2}, \tag{2}$$

where $c(v^2)$ is a normalization coefficient, $\phi^a(x) = 6x(1-x)$ is an asymptotic form of wave function and v is a typical velocity of quark-antiquark pair inside the charmonuim. In accordance to the potential model for J/ψ and η_c mesons $v^2 \approx 0.23$ [13].

The results of these estimations have been compared with the predictions of the δ-approximation ($\phi(x, v^2) = \delta(x - 1/2)$) and with the prediction for the case of "tailles" wave function:

$$\phi_{tailles}(x, v^2) = \begin{cases} 2.5 & 0.3 < x < 0.7 \\ 0 & x < 0.3,\ x > 0.7 \end{cases} \tag{3}$$

The cross section estimations within the model (1) for the wave functions (2) and (3) have been done numerically. The analytical expression for δ-approximation is wellknown:

$$\sigma \sim \frac{(1-4M^2/s)^{3/2}}{s^4}, \tag{4}$$

where s is the e^+e^- interaction energy squared, and M^2 is a charmonium mass.

The ratio between our calculation results and the δ-approximation are shown in fig. 3 for different values of charm quark masses and wave function shapes. The δ-approximation predictions are given for $m_c = 1.5$ GeV.

The toy model under discussion do not allows to shed light on all aspects of double charmonium production, nevertheless the two main features of such process are clearly seen:

1. The cross section value is very sensitive to the c quark mass value at at interaction energies lower than 15 GeV.

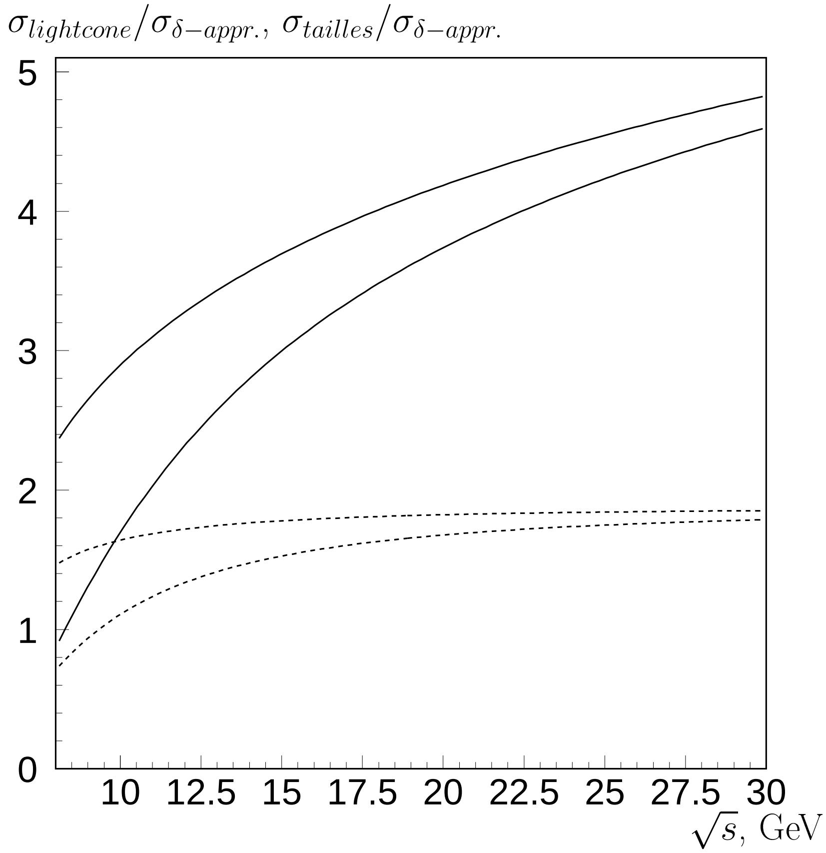

Figure 3: The ratio between the calculation results and the δ-approximation for different values of charm quark masses and wave function shapes as a function of interaction energy: upper solid curve — $m_c = 1.25$ GeV and the lightcone wave function (2); lower solid curve — $m_c = 1.5$ GeV and the lightcone wave function (2); upper dashed curve — $m_c = 1.25$ GeV and the "tailles" wave function (3); lower dashed curve — $m_c = 1.5$ GeV and the "tailless" wave function (3). The δ-approximation predictions are given for $m_c = 1.5$ GeV.

2. The cross section value depends strongly on the wave function behavior on the tails (at $x < 0.3$ and $x > 0.7$).

As one can see from fig. 3 the uncertainties caused by c quark mass value are large at energy 10.6 GeV, where Belle and BaBar data on doubly charmonium production have been received. The cross section value for wave function (2) is increased by factor 1.9 for $m_c = 1.5$ GeV and by factor 3 for $m_c = 1.25$ GeV in comparison to the δ approximation. For the "tailless" wave function (3) the cross section enhancement is about 20% for $m_c = 1.5$ GeV. For $m_c = 1.25$ GeV the cross section value is increased by factor 1.6. Unfortunately the tail behavior is not known with sufficient accuracy. Also it is not clear which c quark mass should be used to obtain the cross section. For the δ-approximation it should be better to put $m_c = M/2$ to obtain the correct meson mass. The situation is not so obvious if one takes into account the quark internal motion, because this motion increases the invariant mass of quark-aniquark system. This a reason to choose the c quark mass value slightly smaller than $M/2$. It is worth to note that for the "massless" c quarks the model under discussion leads the cross section increase by factor 10, and, therefore reproduce approximately the predictions of lightcone formalism.

Contrary to the double charmonium production in e^+e^- annihilation, the internal motion can be neglected for the process of associative charmonium production $e^+e^- \to J/\psi c\bar{c}$. The cross section estimations for this process have been done within the same model, as for the double charmonium production:

$$A_{J/\psi c\bar{c}} = \int dx\, \phi_{J/\psi}(x) T_{c\bar{c}c\bar{c}}. \qquad (5)$$

Our estimations show that the internal motion changes the cross section value by 5-10% in comparison to δ-approximation [14]. These results are in agreement with the predictions [10]. The dependence on c quark mass is more essential for this process.

To summarize:

- Within the model under discussion the cross section value for the process $e^+e^- \to J/\psi\eta_c$ at 10.6 GeV can be increased by factor 1.2-3 depending on the c quark mass value and the wave function shape.

- In the frame of this model the dependence on the wave function shape can be neglected for the process $e^+e^- \to J/\psi c\bar{c}$.

- The additional theoretical studies of charmonium production processes in e^+e^- annihilations are needed.

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